

## Simulation Studies

\*C. Borwankar, N. Bhatt and M.K. Koul

The MACE telescope detects VHE gamma-rays by detecting atmospheric Cherenkov radiation from extensive air showers induced by them. The detection of VHE gamma-rays using IACTs like MACE is hindered by the cosmic rays which continuously bombard the Earth's atmosphere from all directions. These cosmic rays also produce extensive air shower through hadronic interactions with atmospheric nuclei and emit atmospheric Cherenkov radiation. An IACT collects far many more images of cosmic ray induced air showers since cosmic rays outnumber VHE gamma-rays by a factor of  $\sim 1000$ . An IACT extracts gamma-ray signal from overwhelming cosmic ray background using geometrical differences between a gamma-ray induced shower image and a cosmic ray induced shower image. The geometrical features of gamma-ray images are also used to estimate the energy and arrival direction of primary gamma-rays, once they are segregated from background.

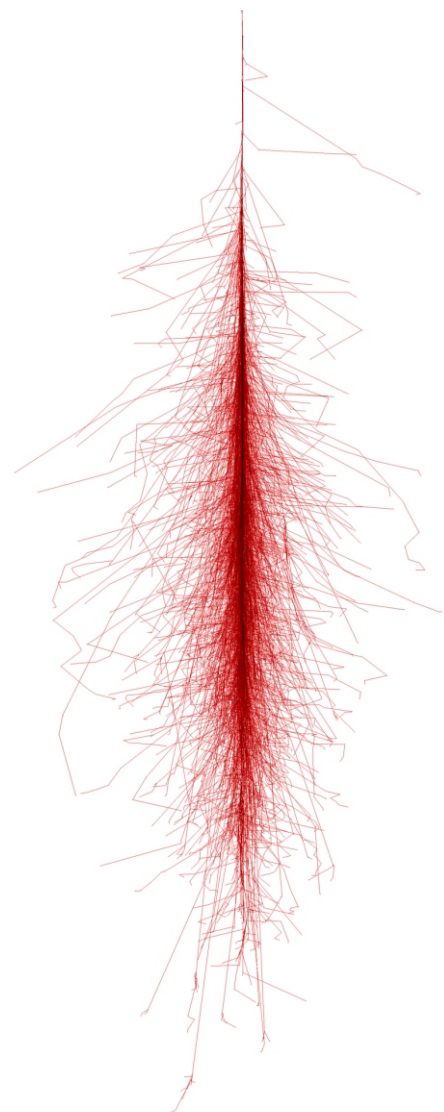
The gamma-ray identification, as well as estimation of energy and arrival direction thus require prior knowledge of image characteristics of gamma-ray and cosmic ray induced shower images. Ideally, such data can be collected by exposing telescope to the calibrated reference beams of gamma/cosmic rays. However due to lack of terrestrial reference beams of VHE gamma/cosmic rays, gamma-ray astronomers rely on simulations.

In an IACT simulation, complete development of gamma/cosmic ray induced extensive air showers, atmospheric Cherenkov emission from them and response of all the components of an IACT to Cherenkov light is simulated. Such simulations then provide calibration data required to extract signal and to determine spectrum and morphology of the gamma-ray source observed. Simulations thus play very crucial role in detection of VHE gamma-rays using IACT.

### Details of MACE Simulation

#### **Simulation of Extensive Air Showers**

We have generated large database of  $\sim 1.2$  billion air showers induced by gamma, proton, electron and alpha particles. The development of extensive air shower through atmosphere is simulated using widely used simulation software CORSIKA[1]. The CORSIKA output contains information about wavelength, direction, position and time of arrival at the observation level for all the secondary particles as well as Cherenkov photons produced in the



▲ A 100GeV GAMMA shower.

extensive air shower. The shower development of primary particles in the energy range of 10 GeV to 20 TeV arriving from all directions within the field of view of MACE are simulated. We have used QGSJET I and FLUKA models to simulate high and low energy hadronic interactions, respectively. We have simulated electromagnetic interactions using EGS4 models. A standard atmospheric density and extinction

coefficient profile as provided by add-on IACT/ATMO package of CORSIKA is used in the simulations. The North and downward components of geomagnetic field at Hanle site are measured to be 31.95  $\mu\text{T}$  and 38.49  $\mu\text{T}$  respectively using IGRF model[2]. Cherenkov radiation within the wavelength range of 240nm-650nm is considered for simulations.

**Simulation of MACE Response**

We have developed C++/ROOT[3] based software for the simulation of MACE response to extensive air shower. The simulation software reads the output data of CORSIKA and then simulates all the components of MACE like the tessellated reflector along with its reflectivity, compound parabolic concentrators, PMT array at focal plane, quantum efficiency of PMTs, effects of light of night sky, formation of images on the focal plane and the electronics related to trigger and data acquisition of MACE.

A parabolic reflector with spherical mirrors of varying focal lengths as deployed in actual MACE reflector are configured in simulation as well. We used measured value of spot sizes to account for distortions caused by misalignment and aberration effects.

The measured values of compound parabolic concentrator efficiency and quantum efficiency are used. To simulate the gain and pulse shape of PMT, we use the ticket data of PMTs as provided by the manufacturer. The values of LONS-flux in different optical bands as reported by Himalayan Chandra Telescope located at Hanle, are used to estimate the charge produced by LONS in each PMT within the charge integration window of telescope. The MACE telescope acquires the charge data in PMTs only when more than 3 neighboring PMTs contemporaneously have pulse amplitudes larger than the mean level of noise induced pulses.

The simulation software simulates PMT pulse amplitude for each PMT periodically at prescribed resolution and detects when the trigger criterion is fulfilled. The simulation software then simulates complete data acquisition electronics consisting of amplification, sampling and digitization stages and saves the data for triggered images.

Complete data analysis procedures are applied on the simulated shower images which pass the trigger criterion to estimate telescope performance as well as to generate calibration data required in data analysis.

**Results of Simulation**

**Effective Area and Energy Threshold**

The Cherenkov radiation from gamma-ray induced extensive air shower forms a circular pool of radius  $\sim 120\text{m}$  with almost constant Cherenkov photon density around the direction of arrival of primary gamma-ray. Beyond 120m, the Cherenkov photon density falls exponentially. Due to significant spread in Cherenkov light from gamma-ray induced shower on ground, an IACT can detect VHE gamma-ray photons falling within the circular area of radius of at least  $\sim 120\text{m}$  around the telescope optical axis. The area of

reflector of gamma-ray is thus not equal to detection area of an IACT. One has to estimate effective detection area when determining the flux of VHE gamma-rays from the observed source. This estimation is performed through simulations of IACT.

To calculate the effective area of an IACT, we first estimate the trigger probability of the telescope as a function of gamma-ray energy and perpendicular distance of gamma-ray axis from the telescope axis. The perpendicular distance between telescope axis and gamma-ray direction is called impact parameter of a gamma-ray induced shower. The trigger probability for a gamma-ray photon of energy E and impact parameter R, is given by

$$p(R,E) = \frac{N_{\text{triggered}}(R,E)}{N_{\text{total}}(R,E)}$$

where  $N_{\text{triggered}}(R,E)$  is the numbered of triggered  $\gamma$  ray events with energy E and impact parameter R, while  $N_{\text{total}}(R,E)$  is the total number of simulated  $\gamma$  ray showers with energy E and impact parameter R. The effective area of an IACT is calculated as a function of energy by integrating the trigger probability over the impact parameters. Thus effective area as a function of energy is given by

$$A(E) = \int_0^{R_{\text{max}}} p(R,E)dR$$

The discrete form of above integral is used to estimate the effective area at various energies in simulation. The effective area is calculated as

$$A(E_j) = \sum_{i=0}^{i=N} p(R_i, E_j) * \pi * (R_{i+1}^2 - R_i^2)$$

where  $E_j$  is the energy in the  $j^{\text{th}}$  energy bin, and  $R_i$  is the impact parameter in the  $i^{\text{th}}$  bin. The gamma-ray showers at higher zenith travel for longer distances in the atmosphere than the vertical showers owing to their oblique direction. This longer traversing length leads to higher attenuation of Cherenkov radiation for high zenith showers.

The attenuation is sufficient to cause reduction in trigger probability at low energies leading to reduction in effective area at low energies. At higher energies, the oblique incidence of the showers creates an elliptical Cherenkov pool on the ground with larger area. At the same time, the Cherenkov yield of high energy showers is sufficient to trigger the telescope even after atmospheric attenuation. As a result, we see that the effective area at low energies decreases with increasing zenith angle while effective area at high energies increases with increasing zenith angle. The energy threshold of MACE i.e., the lowest energy of a gamma-ray which can be detected by the telescope increases with increasing zenith angle. The differential trigger rate is calculated for the point like gamma-ray source with Crab Nebula like spectrum. It is obtained by multiplying the differential flux of the Crab Nebula by effective area.

**Integral Flux Sensitivity**

The IACT data contains overwhelming amount of background events induced by cosmic rays compared to gamma-rays. The gamma-ray shower images are segregated from the cosmic ray images by exploiting differences between various parameters related to shape and orientation of shower images known as Hillas parameters. Gamma-ray events are extracted from cosmic ray

background by applying cuts on Hillas parameters or by using Machine learning models which are trained on the simulated gamma-ray and cosmic ray shower images.

However, none of these methods achieve 100% background rejection along with 100% retention for gamma-ray events. Therefore, detection of a VHE gamma-ray source can only be claimed after statistical analysis. The gamma-ray source is said to be detected by an IACT only when the difference between the number of gamma-like images collected during observations from a gamma-ray source direction and the number of gamma-like images collected during background observation is 5 times the standard deviation of background counts. The background observation consists of observing a patch of sky with no known gamma-ray source. The capability of an IACT to suppress background events while retaining gamma-ray events is measured by integral flux sensitivity of the telescope. The integral flux sensitivity of an IACT is defined as the minimum integral photon flux above an energy threshold which can be detected by an IACT in 50 hours of observations at a statistical significance level of 5. The integral flux sensitivity plays a major role in allocating the IACT observation time to various astrophysical sources since IACTs have small duty cycle and their time must be used judiciously.

The background counts are primarily generated by the cosmic ray air showers. To estimate the integral flux sensitivity one must estimate the mean rate of gamma-like images arising from the cosmic ray induced showers as well as gamma-ray induced showers. This is achieved by performing all the data analysis steps like image cleaning, calculation of Hillas parameters, training the Random Forest classification model for the gamma/cosmic segregation and application of cuts on parameters of simulated images for an IACT. By folding in the respective differential spectra of gamma-ray and cosmic ray particles, the mean rate of

gamma-like events arising from gamma-ray source as well as cosmic ray showers can be estimated. The statistical significance  $\sigma$  of the gamma-ray detection in observation time  $T_{obs}$  is given by

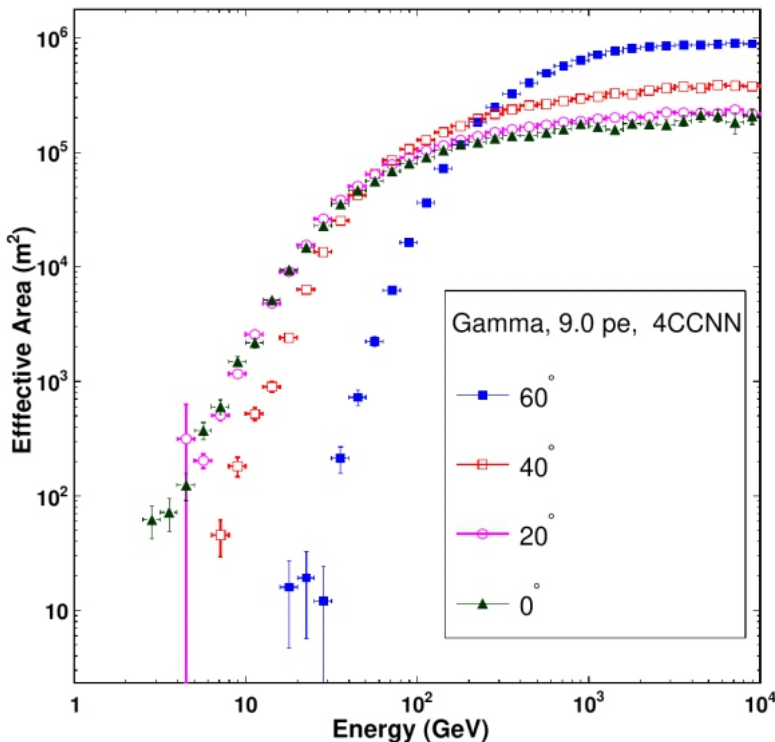
$$\sigma = \frac{R_{\gamma} T_{obs}}{\sqrt{R_{\gamma} T_{obs} + 2R_{CR} T_{obs}}}$$

where  $R_{\gamma}$  is the gamma-ray rate after folding in efficiency of gamma/hadron segregation into trigger rates, while  $R_{CR}$  is the rate of gamma-like events from cosmic ray induced showers obtained after folding in efficiency of gamma/hadron cuts into cosmic ray trigger rates.  $R_{\gamma}$  and  $R_{CR}$  depend on the differential spectra of the gamma-rays and cosmic rays. Measured values of cosmic ray spectra for different constituent particles are used to estimate  $R_{CR}$ .  $R_{\gamma}$  is estimated using spectrum of Crab Nebula which is the steady source of VHE gamma-rays. The integral flux sensitivity as a function of energy threshold for a Crab like point source is presented in this article. The blue dashed line with triangular points shows the average integral flux sensitivity of the MACE telescope in the zenith angle range of  $0^{\circ}$  to  $30^{\circ}$ . The red line shows the integral flux sensitivity as a function of energy threshold at the zenith angle of  $40^{\circ}$ . The integral flux of Crab Nebula is shown by black dashed line. The MACE telescope can detect point gamma-ray sources with integral flux of  $\sim 2.4\%$  of Crab Nebula above an energy threshold of  $\sim 30$  GeV in 50 hours of observation.

#### Angular and Energy Resolution of MACE

The arrival direction of detected gamma-rays can be determined by an IACT telescope using the correlation between Hillas parameters and angular position of the primary gamma-ray in the camera plane. The elongation of a gamma-ray induced shower image increases with increasing offset of primary gamma-ray direction with respect to the telescope optical axis. The ratio of the Width and Length parameters of the gamma-ray shower image can be used to quantify the elongation of shower images. Simulated gamma-ray images provide the database to estimate the correlation between image elongation and source position in the camera plane.

We used Random Forest regression model trained on the simulated database to estimate the source position of the gamma-ray in camera plane, from the image parameters. The trained Random Forest models are also used to estimate the angular resolution of the telescope at various energies. The angular resolution of the telescope is the ability to resolve two nearby gamma-ray point sources. It is quantified by estimating standard deviation of the 2-dimensional Gaussian function, fitted to distribution of reconstructed arrival directions of simulated gamma-rays in the camera plane. The variation of angular resolution as a function of energy in the zenith angle range of  $0^{\circ}$  to  $30^{\circ}$  and at the zenith angle of  $40^{\circ}$  is given in this article. The MACE angular resolution varies from  $\sim 0.21^{\circ}$  near the energy threshold to  $\sim 0.07^{\circ}$  at energies above 1 TeV. The total charge content and Length of the gamma-ray induced shower images are directly correlated with the energy of the primary gamma-ray. This correlation is used to reconstruct the energy of the detected gamma-ray. An IACT simulation also provides the energy resolution of the telescope at various energies. It is defined as the



▲ EFFECTIVE AREA of the MACE telescope as a function of energy at zenith angles of  $0^{\circ}$ ,  $20^{\circ}$ ,  $40^{\circ}$  and  $60^{\circ}$ .

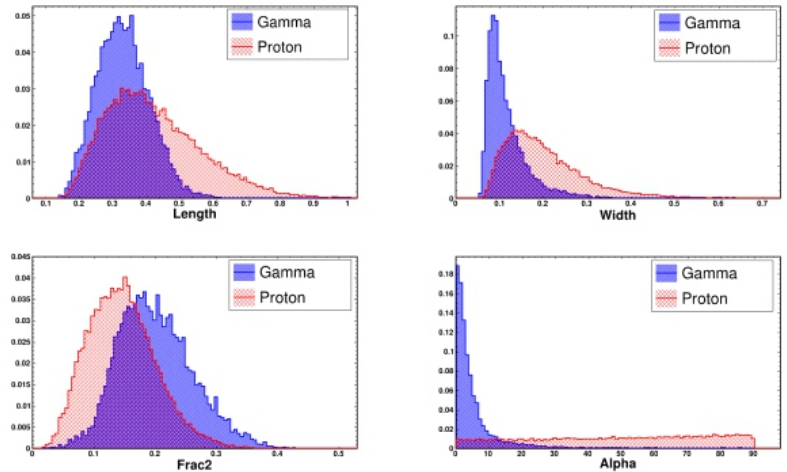
standard deviation of the fractional difference between the true energy and estimated energy of gamma-ray events. MACE energy resolution varies from ~40% near its threshold to ~19.7% at energies above 1 TeV.

**Conclusion**

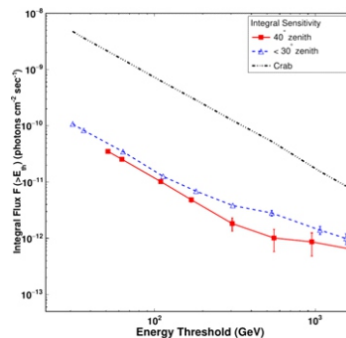
Simulations of an IACT are essential part of an its operations and data analysis. They play crucial role of providing calibration data related to gamma/hadron segregation efficiency, effective detection area, angular and energy resolution of the telescope. Detailed simulation studies involving simulation of shower development using CORSIKA and simulation of all sub components of the MACE telescope have been carried out. We have estimated effective area and trigger energy threshold, integral flux sensitivity, angular resolution and energy resolution of the MACE telescope at various zenith angles between 0° to 40°. MACE is able to detect weak gamma-ray sources with integral flux of ~2.4% of the Crab Nebula in the zenith range of 0° to 40°. The energy threshold of MACE is estimated to be between ~30 GeV to ~50 GeV for this zenith range. The angular resolution of the MACE telescope is ~0.21° near the energy threshold while at high energies of > 1 TeV the angular resolution is ~0.06°. The energy resolution of MACE is expected to improve from ~40% near the threshold to ~20% at energies above 1 TeV.

**Bibliography**

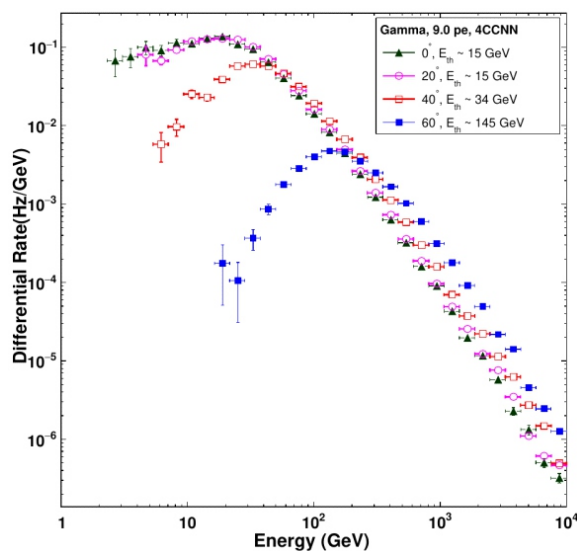
[1] D. Heck et al., *Astro-physics Source Code Library*, 2012.  
 [2] E. Thebault et al., *Earth, Planets, and Space*, 2015, 81, 67.  
 [3] R. Brun and F. Rademakers, *Nuclear Instruments and Methods in Physics Research A*, 1997, 389, 81.



▲ DISTRIBUTION of Length, Width, Alpha and Frac2 parameters for gamma-ray induced air shower images (Blue) and proton induced shower images (Red). Frac2 is the ratio of total charge content in 2 brightest pixels to the total charge content in full image.

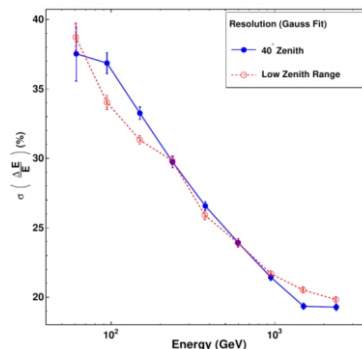
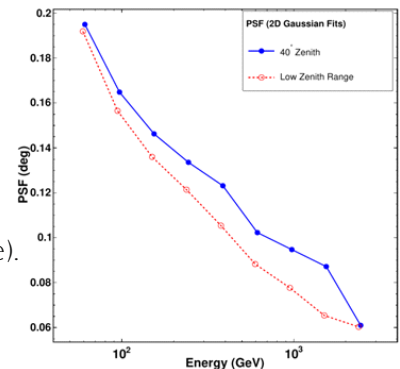


◀ INTEGRAL FLUX sensitivity of Crab nebula in zenith angle range of 0° to 30° (blue curve) and 40° (red line).



▲ DIFFERENTIAL RATE curves for gamma-rays at various zenith angles of 0°, 20°, 40° and 60°. The energy threshold, defined as the position of the peak of the differential rate curve for gamma-rays, changes from ~15 GeV at 0° zenith to ~143 GeV at 60°.

▶ ANGULAR RESOLUTION of the MACE telescope in the zenith angle range of 0°-30° (red) and at the zenith angle of 40° (blue).



◀ ENERGY RESOLUTION of the MACE telescope in the zenith angle range of 0°-30° (red) and at the zenith angle of 40° (blue).

The authors are from Astrophysical Sciences Division of Bhabha Atomic Research Centre, Trombay, Mumbai.

✉ chinmay@barc.gov.in\*