NON-DESTRUCTIVE ASSAY OF NUCLEAR WASTE BARRELS USING ACTIVE AND PASSIVE COMPUTED TOMOGRAPHY

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Introduction

Assay of nuclear waste drums is required for decisions on its disposal, safe transportation, permanent storage as well as nuclear material accounting. Manual inspection of opened drums for an assay is a risky, time-consuming, and expensive proposition. Traditionally, the drums are inspected by conventional transmission radiography or tomography. However, these techniques do not yield information about the radioisotopes inside the drum, but just on the material density. Gamma spectroscopy in open geometry or segmented gamma scanning can be performed to this aim, but to improve accuracy and provide information about the spatial distribution of the radioisotopes, an active and passive computed tomography (A&PCT) technique is best suited.

Principle of Active & Passive CT

Active and Passive Computed Tomography (A&PCT) is one of the most efficient techniques for characterization (source strength estimation) and localization of radioisotopes in nuclear waste assay. It uses the decay of radioactive isotopes to image the spatial distribution of the isotope as well as to determine their source strength or activity. The A&PCT method consists of two steps to perform an assay: Active CT and Passive CT.

![Fig.1: Schematic of A&PCT set-up for waste assay](image)
In active CT, attenuation map of the object is obtained. This is similar to conventional X-ray CT but it uses an external gamma source (instead of X-ray) and the spectrum is recorded using multi channel analyzer (MCA). It differs from conventional CT scanners in that it discriminates between photons of different energies. The gamma source used for active CT has, generally, multiple emission energies. The reconstruction results are a discrete quantitative measurement of the linear attenuation coefficient at each energy measured, i.e., there has been no integration over the energy spectrum. Thus active CT images have pixels that represent the absolute measurement that denote the attenuation map of the surrounding object.

Passive CT is used to measure and determine the location, identity, and strength of radioisotope sources within an object. The ray sum for passive CT (also called Single Photon Emission Computed Tomography (SPECT)) is the counts measured in disintegrations per unit volume per unit time of the passive source within the object. Therefore, a single-photon-emitted ray sum is the integrated radioisotope activity, modified by one or multiple of exponential attenuations, along the path from a source position within the object to the detector. The function that is imaged for passive CT is the measured gamma-ray activity at one or more energies of all detectable radioisotopes within the object. By combining active and passive measurements, corrections can be made to account for the effect that the waste contents have on the internal radioactive emissions. The corrected gamma-ray spectra can be used to identify, localize, and assay all measured radioisotopes present in the container.

Mathematics of SPECT
Consider a vector \( \mathbf{x} = (x, y) \) in a two-dimensional Euclidean space (Fig.2). Let \( f(\mathbf{x}) \) denote the distribution of radioisotope activity and \( \mu(\mathbf{x}) \) denote the attenuation map of the surrounding object.

![Fig.2: A typical SPECT set-up in parallel geometry](image-url)
The attenuated 2D Radon transform at energy $E$ for parallel beam geometry is of the form\[1\]
\[
\mathbb{R}_{ab}(s, \theta; E) = g(s, \theta; E) = \int_{-\infty}^{\infty} f(s\theta + t\frac{\sin \theta}{\cos \theta}; E) e^{-\int_{0}^{\infty} \mu(\bar{x} + p\varphi; E) dp} dt
\]
where $\bar{\theta} = \begin{pmatrix} \cos \theta \\ -\sin \theta \end{pmatrix}$, $\varphi = \begin{pmatrix} \cos \varphi \\ \sin \varphi \end{pmatrix}$ and $D_{\mu}(\bar{x}, \varphi; E)$ is the divergent beam transform of $\mu(\bar{x}; E)$ in the direction of $\varphi$. Defined as $D_{\mu}(\bar{x}, \varphi; E) = \int_{0}^{\infty} \mu(\bar{x} + p\varphi; E) dp$

In matrix notation, the SPECT problem can be written as

\[
g = Af
\]

or,

\[
\begin{pmatrix}
ge_1 \\
ge_2 \\
. \\
ge_m
\end{pmatrix} =
\begin{pmatrix}
a_{11} & a_{12} & \ldots & a_{1N} \\
a_{21} & \ldots & \ldots & \ldots \\
. & \ldots & \ldots & \ldots \\
a_{m1} & \ldots & a_{m1} & \ldots & a_{mN}
\end{pmatrix}
\begin{pmatrix}
f_1 \\
f_2 \\
. \\
f_m
\end{pmatrix}
\]

where $g \in \mathbb{R}^{m \times 1}$ is the observed projection data, $A \in \mathbb{R}^{m \times N}$ is the probability system matrix and $f \in \mathbb{R}^{N \times 1}$ is the vector of unknown spatial density distribution of nuclear disintegration events resulting in gamma emission in the object. The system model can take into account various physical factors involved in the detection process – attenuation, collimation, scatter, distance-dependent fall in intensity, transmission through collimator, etc. All these factors can be modeled into the system matrix. The attenuation factor is modeled using the attenuation map obtained in the active CT step. The problem thus reduces to solving the linear system of equations (2).

**Reconstruction Algorithms**

Mathematically, reconstruction is an inverse problem: the aim is to find vector of unknowns from the measured observables by solving a system of linear equations. Reconstruction algorithms generally employed for solving SPECT problem can be divided into two classes: Analytical and Iterative Reconstruction Techniques.

The analytical approach assumes noiseless data, an ideal collimator, no attenuation and no scatter of gamma radiation. These assumptions can make the reconstruction result an inaccurate representation of the true activity distribution. Therefore, additional data filtering and post-processing are necessary. The most commonly used method is the Filtered Back Projection
(FBP) algorithm. In this approach, the projections are filtered in the frequency domain and the filtered projections are backprojected in the spatial domain. An analytical solution of Eq.1 was first provided by Novikov [11] as follows:

\[
f(x) = \frac{1}{4\pi} \text{Re}\left\{ \int_0^{2\pi} \nabla \left( e^{\frac{i}{2} (I + i\mathcal{H}) [Re\mu] (s,\theta)} \mathcal{H} \mathcal{e}^{\frac{i}{2} (I + i\mathcal{H}) (s,\theta)} g(s,\theta) \right) |_{\theta = \pi} d\theta \right\}
\]

where \( h(s,\theta) = \frac{1}{2} (I + i\mathcal{H}) [Re\mu] (s,\theta) \) and \( \mathcal{H} \mathcal{e}\phi(s,\theta) = \frac{1}{\pi} \text{p.v.} \int_{-\infty}^{\infty} \frac{\phi(s',\theta)}{s-s'} ds' \)

with \( i^2 = -1 \). \( I \) is the identity operator. \( \mathcal{H} \) is the Hilbert transform with respect to the second parameter and \( \text{p.v.} \) denotes Cauchy principal value of the integral. \( \nabla \) is the divergence operator.

The iterative approach [79] allows a complex model of gamma radiation interaction to be taken into account including the effect of collimation and scatter as well as statistical variability of measured data. These are recommended for quantitative image analysis which is crucial, for example, in waste assay. The algebraic reconstruction techniques include simple ART, SART, SIRT, etc. The statistical reconstruction technique takes into account the Poisson statistics of radiation. The techniques include MLEM, OSEM, MAP-EM, etc.

One of the commonly used iterative techniques for reconstruction of Passive CT data is Maximum Likelihood Expectation Maximization (MLEM) technique. The MLEM formulation is mathematically stated as [21]:

\[
\hat{f}^{k+1}_j = \frac{\hat{f}^k_j}{\sum_{i=1}^{M} a_{ij}} \sum_{i=1}^{M} g_i \sum_{j'=1}^{N} a_{ij'} \hat{f}^k_{j'}
\]

**Experimental set-up**

In order to demonstrate the capability of doing active and passive CT, an experimental system has been set-up at Purnima Labs (Fig.3). It consists of the following:

- Sample Stage
- Sample / object matrix with (passive) source
- Detector and associated electronics
- Collimator
- Data Acquisition system
- (External) Active source (for transmission measurement)

For the experiments, a mock-up waste drum (580mm diameter and 850mm height) filled with cotton waste/gloves/tissues is used and radionuclide samples are placed at different locations inside the drum.
Experimental Results

Various experiments have been carried out using this set-up. Some of the results are presented below:

(a) Parallel Beam Geometry

Fig. 4(a) shows the reconstructed TCT data for three slices at different energies.

Fig. 4: (a) Active CT data for three different slices (top to bottom) at different energies (from left to right) – 121 keV, 244 keV, 344 keV, 444 keV, 778 keV (b) 3D view of reconstructed $^{137}$Cs activity

Three $^{137}$Cs sources are placed inside the drum at different locations. Transmission computed tomography (TCT) of the drum is carried out using external $^{152}$Eu source. The TCT data are acquired for different energies (121 keV, 244 keV, 344 keV, 444 keV, 778 keV and 867 keV). The attenuation map is reconstructed for each energy using Filtered Backprojection algorithm. Fig. 4(a) shows the reconstructed TCT data for three slices at different energies.
The attenuation map for 662 keV (emission energy in SPECT) is interpolated from these data. This attenuation map is then used for attenuation compensation. For Passive CT, projections are acquired with LaBr$_3$(Ce) detectors at 36 angular locations and 24 lateral positions for each z-position. In the vertical direction, 34 z-positions are scanned. Reconstruction is done using Filtered Backprojection scheme with attenuation compensation based on Novikov’s Inversion Formula. The reconstructed 3D volume is shown in Fig. 4(b).

(b) Fan Beam Geometry
The schematic experimental arrangement for fan-beam geometry is shown in Fig. 5. It has the advantage of performing Active CT with single active source and multiple detectors.

![Fan Beam Geometry Diagram]

Fig. 5: Schematic arrangement for multiple detector A&PCT scan in Fan beam configurations

Fig. 6(a) shows the attenuation map of drum contents. Fig. 6(b) & 6(c) show the reconstructed 137Cs activity using Analytical and 2D MLEM techniques respectively.

![Reconstructed Attenuation Maps]

Fig. 6 (a): Reconstructed attenuation map (b) & (c) Analytical and 2D MLEM reconstructed volume showing $^{137}\text{Cs}$ activity respectively
(c) Imaging of $^{239}$Pu

Experiments were performed for imaging of $^{239}$Pu in waste drums using A&PCT technique.

For the ACT measurement, the external gamma source and a single LaBr$_3$(Ce) detector are used. The attenuation map (at 414 keV) is shown in Fig.7(a). For the PCT measurement, the external source is removed and gamma rays emitted from within the drum are recorded by three collimated LaBr$_3$(Ce) detectors. Data is acquired at 24 lateral positions and 12 angular positions over 360° for each z-position. In the vertical direction, 33 z-positions were scanned. Each slice thickness is 25mm. The 414 keV peak from $^{239}$Pu is used for PCT reconstruction. Fig.10 (b) shows the reconstructed 3D volume. The radioisotope activity distribution can clearly be observed.

(d) Fully 3D Reconstruction

For 2D reconstruction, system matrix is constructed for a single slice and the corresponding slice is reconstructed. The 3D activity map is obtained by vertically stacking the respective 2D reconstructed slices. It is assumed that all photon counts in a particular row of the planar image are due to photons emitted from the slice at same height. However, the projection data has contribution from other voxels also which are located in other slices. In fact, each detector or projection bin accepts contributions from a cone whose dimensions depend on the shape and size of collimator holes.

For fully 3D reconstruction, 3D system matrix corresponding to 3D collimator is generated for the whole object. This method requires large storage space for matrices, is computationally quite
intensive and much harder to implement. However, fully 3D reconstruction should reduce both intra-slice and inter-slice blurring.

Fig. 8 (a), (b) and (c) show the 3D view of $^{137}$Cs activity distribution using analytical, 2D MLEM and fully 3D MLEM reconstruction respectively (for the sample in Fig.4). The effect of collimator blurring is clearly evident from these images. For fully 3D MLEM reconstruction, (Fig. 8(c)), both inter- and intra-slice blurring are reduced considerably and the point source appear point-like.

Conclusion

The development of an active and passive gamma emission tomography technique has been reported. This technique can be used for non-destructive assay of nuclear waste barrels. Some experimental results with $^{137}$Cs as well as $^{239}$Pu sources have been presented.

References


